Quiz 3-B [16 marks]

1. [Maximum mark: 8] 25M.1.AHL.TZ3.8

Consider the complex number $z_1 = \sqrt{3} - 3\mathrm{i}$.

(a) Express z_1 in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$. [3]

Markscheme

attempt to find modulus **OR** argument of z_1 ***(M1)***

$r = \sqrt{12}\ \left(= 2\sqrt{3}\right)$ or $\theta = -\frac{\pi}{3}$ ***A1***

$z_1 = \sqrt{12}\mathrm{e}^{-\mathrm{i}\frac{\pi}{3}} \left(= 2\sqrt{3}\mathrm{e}^{-\mathrm{i}\frac{\pi}{3}}\right)$ ***A1***

[3 marks]

Consider the complex number $z_2 = 2\sqrt{3}\mathrm{e}^{\mathrm{i}\frac{5\pi}{6}}$.

The cube roots of $\frac{z_2}{z_1}$ are denoted by u, v and w.

(b) Find u, v and w. [5]

Markscheme

$\left(\frac{z_2}{z_1} = \frac{2\sqrt{3}\mathrm{e}^{\mathrm{i}\frac{5\pi}{6}}}{2\sqrt{3}\mathrm{e}^{-\mathrm{i}\frac{\pi}{3}}} =\right) \mathrm{e}^{\mathrm{i}\frac{7\pi}{6}} \left(= \mathrm{e}^{-\mathrm{i}\frac{5\pi}{6}}\right)$ ***(A1)***

recognizing three equivalent arguments ***(M1)*** eg $\frac{z_2}{z_1} = \mathrm{e}^{\mathrm{i}\left(-\frac{5\pi}{6}+2\pi k\right)}$ **OR** $\frac{z_2}{z_1} = \mathrm{e}^{-\frac{5\pi}{6}\mathrm{i}},\ \mathrm{e}^{-\frac{17\pi}{6}\mathrm{i}},\ \mathrm{e}^{\frac{7\pi}{6}\mathrm{i}}$

attempt to find a root using de Moivre's theorem ***(M1)***

$\mathrm{e}^{\mathrm{i}\left(-\frac{5\pi}{18}+\frac{2\pi k}{3}\right)} \left(= \mathrm{e}^{\mathrm{i}\left(\frac{(12k-5)\pi}{18}\right)} = \mathrm{e}^{\mathrm{i}\left(\frac{(12k+7)\pi}{18}\right)}\right)$ ***(A1)***

$\mathrm{e}^{-\mathrm{i}\frac{17\pi}{18}},\ \mathrm{e}^{-\mathrm{i}\frac{5\pi}{18}},\ \mathrm{e}^{\mathrm{i}\frac{7\pi}{18}}$ ***A1***

[5 marks]

2. [Maximum mark: 8] 21M.1.AHL.TZ1.7

Consider the quartic equation

$z^4 + 4z^3 + 8z^2 + 80z + 400 = 0,\ z \in \mathbb{C}.$

Two of the roots of this equation are $a + b\mathrm{i}$ and $b + a\mathrm{i}$, where $a,\ b \in \mathbb{Z}$.

Find the possible values of a. [8]

Markscheme

METHOD 1

other two roots are $a - b\mathrm{i}$ and $b - a\mathrm{i}$ ***A1***

sum of roots $= -4$ and product of roots $= 400$ ***A1***

attempt to set sum of four roots equal to -4 or 4 OR
attempt to set product of four roots equal to 400 ***M1***

$a + b\mathrm{i} + a - b\mathrm{i} + b + a\mathrm{i} + b - ai = -4$

$2a+2b=-4(\Rightarrow a+b=-2)$ ***A1***

$(a+b\mathrm{i})(a-b\mathrm{i})\ (b+a\mathrm{i})(b-a\mathrm{i})=400$

$\left(a^2+b^2\right)^2=400$ ***A1***

$a^2+b^2=20$

attempt to solve simultaneous equations ***(M1)***

$a=2$ or $a=-4$ ***A1A1***

METHOD 2

other two roots are $a-b\mathrm{i}$ and $b-a\mathrm{i}$ ***A1***

$(z-(a+b\mathrm{i}))(z-(a-b\mathrm{i}))(z-(b+a\mathrm{i}))(z-(b-a\mathrm{i}))(=0)$ ***A1***

$\left((z-a)^2+b^2\right)\left((z-b)^2+a^2\right)(=0)$

$\left(z^2-2az+a^2+b^2\right)\left(z^2-2bz+b^2+a^2\right)(=0)$ ***A1***

Attempt to equate coefficient of z^3 and constant with the given quartic equation ***M1***

$-2a-2b=4$ and $\left(a^2+b^2\right)^2=400$ ***A1***

attempt to solve simultaneous equations ***(M1)***

$a=2$ or $a=-4$ ***A1A1***

[8 marks]