Chapter Fouir

Discrete Probability Distributions

Introduction to Probability Distributions

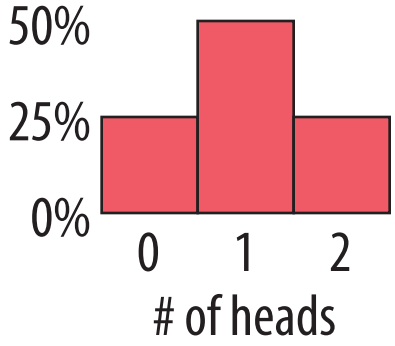
Geometric and Binomial Probabilities

Binomial Distributions

Probability Distributions

A **probability distribution** states each possible outcome or range of outcomes of an event and how likely it is.

A probability distribution can be displayed in many ways, such as a sentence, table, or graph. When the variable is numerical, a histogram is commonly used.

Flip two coins.	Sentence	Table	Graph
Probability distribution for number of heads	There is a 25% chance of getting 0 heads, a 50% chance of getting exactly 1 head, and a 25% chance of getting 2 heads.	Probability of exactly *r* heads out of 2 coin flips *r* | *P*(*r*) 0 | 25% 1 | 50% 2 | 25%	Probability of exactly *r* heads out of 2 coin flips 

Discrete and Continuous Variables

Variables for which specific values are counted are **discrete**. Variables for which values are sorted into ranges are **continuous**.

Variable type	Definition	In a histogram	Examples
Discrete	There exist values between which no other values of the variable are possible.	There is a bar for each possible value. Each bar is labeled.	• iPhone capacity (256GB, 512GB, 1TB, etc. but not 400GB or other values in between) • number of people in a kitchen (0, 1, 2, 3, . . ., but not 2¾ or other values in between)
Continuous	There are infinitely many possible values of the variable between any two values of it.	Each bar represents a range of values. The boundaries of the bars are labeled instead of the bars themselves.	• book weight (0 – 100 grams, 100 – 200 grams, . . .) • tree height (0 – 2 meters, 2 – 4 meters, . . .)

Usually for distributions with more than a handful of possible values, the values are grouped into ranges and treated as continuous even if they are technically discrete, such as number of people at a school.

Mean and Standard Deviation of Probability Distributions

The mean of a probability distribution is its expected value, which is the same as its weighted average.

The standard deviation is calculated the same as it is for grouped data, except that the probabilities $P(x)$ are used instead of the frequencies f. Since the sum of the probabilities must be 1, the mean is $\sum xP(x) \div 1$, that is $\mu = \sum xP(x)$.

In the example below, a deck has six white cards worth 0 points, three green cards worth 10 points, and one red card worth 25 points.

Event	x	$P(x)$	$xP(x)$	$x - \mu$	$(x - \mu)^2$	$P(x)(x - \mu)^2$
white card	0	$\frac{6}{10}$	0.0	-5.5	30.25	18.15
green card	10	$\frac{3}{10}$	3.0	4.5	20.25	6.08
red card	25	$\frac{1}{10}$	2.5	19.5	380.25	38.03
TOTAL		1.00	5.5			$\sigma^2 = 62.26$

$$\sigma = \sqrt{62.26} \approx 7.89$$

Binomial Probabilities

A **binomial experiment** is a scenario in which a specific independent event is attempted multiple times so see how many successes there are. The probability of exactly r successes is $P(r) = \binom{n}{r}p^r q^{n-r}$.

Value	Meaning	Example: 3 correct predictions in ten 6-sided die rolls
n	number of trials	10 rolls were made.
r	number of successes	3 rolls were correctly predicted.
p	probability of success on each individual trial	Each roll had a $\frac{1}{6}$ chance of being correctly predicted.
q	probability of failure on each individual trial ($q = p'$)	Each roll had a $\frac{5}{6}$ chance of being incorrectly predicted.
p^r	probability of r successes out of r trials	There is a $(\frac{1}{6})^3 = \frac{1}{216}$ chance of three out of three rolls being correctly predicted.
q^{n-r}	probability of $n - r$ failures out of n trials	There is a $(\frac{5}{6})^7 = \frac{78125}{279936}$ chance of seven out of seven rolls being incorrectly predicted.
$\binom{n}{r}$	number of possible orders of r successes out of n total trials	There are $\binom{10}{3} = 120$ different ways to choose which 3 of the 10 rolls were correctly predicted.
$\binom{n}{r}p^r q^{n-r}$	probability of exactly r successes (and $n - r$ failures) out of n trials	The probability of correctly predicting exactly 3 out of 10 rolls on a 6-sided die is $\binom{10}{3}(\frac{1}{6})^3(\frac{5}{6})^7 = 120(\frac{1}{216})(\frac{78125}{279936}) = \frac{9375000}{60466176} \approx 15.5\%$

Binomial Calculator Functions

The probability of exactly r successes in a binomial experiment, $\binom{n}{r}p^r q^{n-r}$, can be found on the calculator with the `binompdf` function.

The probability of at most r successes in a binomial experiment can be found on the calculator with the `binomcdf` function.

The probability of at least r successes in a binomial experiment can be found on the calculator by taking the complement of at most $r - 1$ successes.

The binomial functions can be selected from the `DISTR` menu.

Function	Probability of…	Example: Roll a die 8 times
`binompdf(n,p,r)`	exactly r successes	Exactly 2 sixes: `binompdf(8,1/6,2)` ≈ `26.0%`
`binomcdf(n,p,r)`	at most r successes	At most 2 sixes: `binomcdf(8,1/6,2)` ≈ `86.5%`
`1–binomcdf(n,p,r–1)`	at least r successes	At least 2 sixes: `1–binomcdf(8,1/6,1)` ≈ `39.5%`

Mean and Standard Deviation of Binomial Distributions

For binomial probability distributions, the calculations for mean and standard deviation are greatly simplified. In addition, the mean is also the most likely outcome, or between the two most likely outcomes if the mean itself is not a whole number.

Statistic	Formula	Example for number of 6's out of 12 dice
Mean	$\mu = np$	$\mu = 12\left(\frac{1}{6}\right) = 2$
Standard Deviation	$\sigma = \sqrt{npq}$	$\sigma = \sqrt{12\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)} \approx 1.29$